Universal n^{-5} spectrum of zonal flows on giant planets

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The energy spectra of the observed zonal flows on Jupiter and Saturn are shown to obey the scaling law $E_Z(n)=C_Z(\Omega/R)^2 n^{-5}$ in the range of total wave numbers n not affected by large scale friction (here, Ω and R are the rotation rate and the radius of the planet, and C_Z is an order-one constant). These spectra broadly resemble their counterpart in recent simulations of turbulent flows on the surface of a rotating sphere [Huang *et al.*, Phys. Fluids **13**, 225 (2001)] that represents a strongly anisotropic flow regime evoked by the planetary vorticity gradient. It is conjectured that this regime governs the large scale circulations and the multiple zonal jets on giant planets. The observed strong equatorial jets that were not produced in the nearly inviscid simulations by Huang *et al.* are attributed to the combined effect of the energy condensation in the lowest zonal modes and the large scale friction. [DOI: 10.1063/1.1373684]

The large scale circulation on giant planets is characterized by quasisteady zonal jets with superimposed coherent vortices. The explanation of the physical mechanism that governs the generation and maintenance of the zonal flows has been one of the major problems of the planetary sciences.[1,2] The zonal circulation has been studied in computer models (see Ref. 3, and references therein) and attributed to the interplay between planetary rotation and the inverse energy cascade inherent to 2D turbulence. Such flows develop anisotropic energy transfer that overwhelmingly prefers the zonal direction. The planetary vorticity gradient also increases stability of zonal flows, enabling them to retain augmented amounts of energy. The small scale forcing necessary to maintain the inverse cascade can be related to internal heat sources present on every giant planet but Uranus.[4] For Jupiter, Ingersoll *et al.*[5] have identified this forcing with small scale convective cells.[6] A distinctive feature of giant planets' atmospheric circulations is a very long time scale of large scale friction, τ,[1,7] such that the wave numbers characterizing the large-scale friction, n_{fr}, and small-scale forcing are separated by one or two decades in spectral space. Thus, there exists a wide range of intermediate wave numbers in which the flow is nearly inviscid and governed by the universal dynamics of strongly rotating turbulence. This parameter range has been explored in our recent numerical simulations of 2D turbulence on the surface of a rotating sphere[3] incorporating a scale selective, high order inverse Laplacian friction with a small and predetermined n_{fr}. A small scale, random, isotropic forcing initiated the flow from rest and maintained its evolution. After long integration, the ensuing flow regime was distinguished by strong alternating zonal jets and highly anisotropic energy spectrum. For $n>n_{\mathrm{fr}}$ and almost all directions in the wave number space, the energy spectrum preserves its Kolmogorov shape,

$$E(n)=C_K\epsilon^{2/3}n^{-5/3}, \tag{1}$$

though with somewhat smaller Kolmogorov–Kraichnan constant, $C_K\simeq 4$, than in nonrotating 2D turbulence (ϵ is the small scale energy injection rate). For the zonal components, a new, steep spectrum,

$$E_Z(n)=C_Z(\Omega/R)^2 n^{-5}, \tag{2}$$

was discovered with the value of C_Z between 0.3 and 0.5. For $n<n_{\mathrm{fr}}$, the spectra quickly fell to zero due to effective energy removal by friction. The spectra in Eqs. (1) and (2) are defined in the same manner as in Huang *et al.*,[3] with n being the total wave number in spherical geometry corresponding to the lower index of a spherical harmonic Y_n^m. The spectrum $E_Z(n)$ contains only zonal components, $m=0$, of the flow field. Huang *et al.*[3] have speculated that a quasisteady equilibrium flow regime with the anisotropic spectrum (2) could exist on the giant planets giving rise to their observed zonal flow structure and very slow variability of large scale circulation. Following this insight, the present study probes the energy spectra of the zonal flows on Jupiter and Saturn obtained from Voyager 1 and 2 to find out whether or not this spectrum is indeed present in the data.

Figures 1(a) and 1(b) show the Voyager median zonal wind profiles for Jupiter and Saturn. The Jupiter wind data[8] was averaged in bins of approximately 0.25° between roughly 60° N and 55° S. The Saturn wind data[9] was averaged in bins of 0.5°–0.6° between roughly 80° N and 70° S. The unobserved polar regions in both data were filled with

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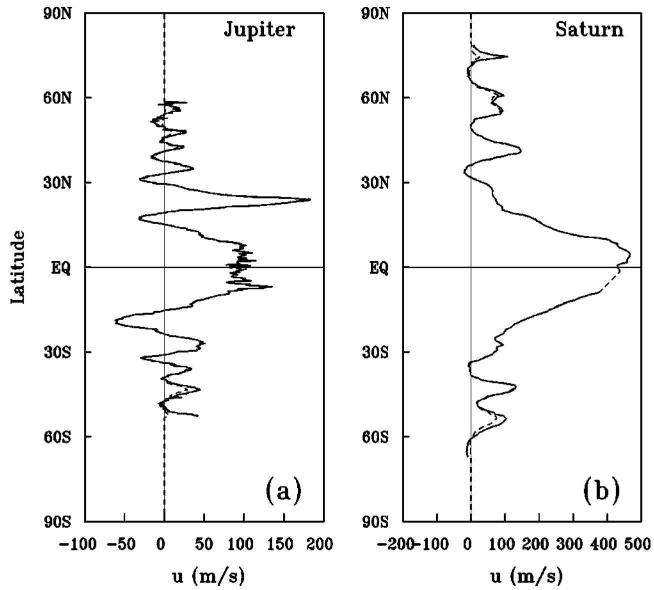

FIG. 1. Zonal winds on Jupiter (a) and Saturn (b) obtained from Voyager 1 and 2. The dashed lines show the tapered profiles that were used in the spectral analysis.

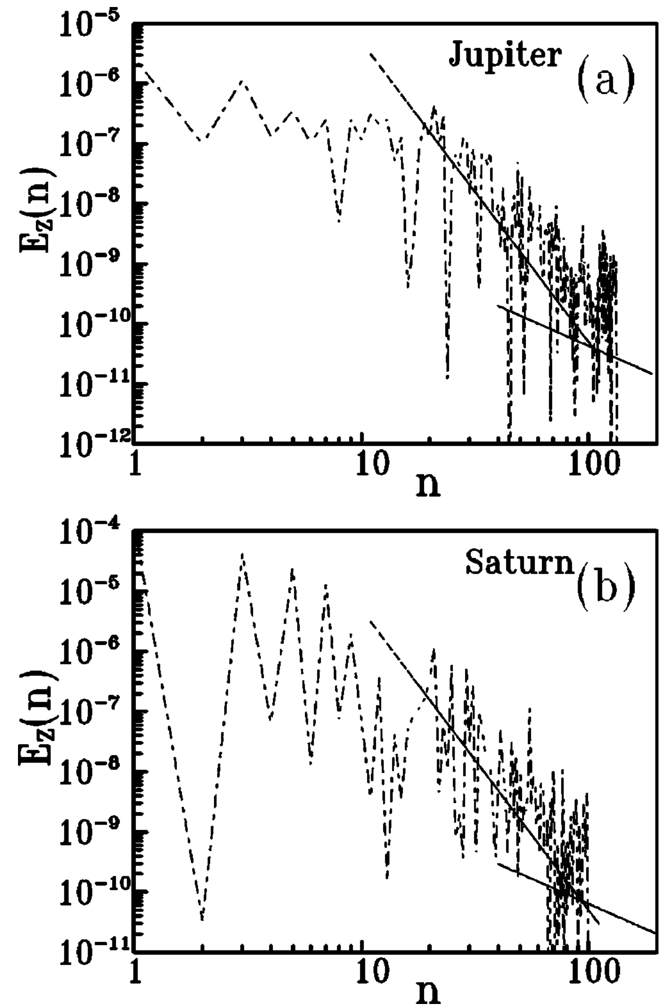

FIG. 2. Observed zonal energy spectra for Jupiter (a) and Saturn (b) normalized with $(\Omega R)^2$. The long and short solid lines are the -5 and Kolmogorov spectra calculated for actual Jupiter and Saturn parameters.

zeros. A smoother was applied to the edges of the data-free regions to remove discontinuities. Another data-free zone between 1° S and 10° S of Saturn, due to obstruction by the ring, was filled by interpolation. The tapered and interpolated profiles, shown in Figs. 1(a) and 1(b) as the dashed curves, were then used to evaluate the energy spectra.

Figures 2(a) and 2(b) show the observed zonal energy spectra for Jupiter and Saturn. The data appear quite noisy which is expected from a single realization of a slowly evolving turbulent flow field. A satisfactory statistical smoothing of the spectra would require extremely long observations. In contrast, in our numerical simulations[3] the smoothing of the spectrum was achieved by ensemble-averaging many independent realizations. In the data analysis below, R and Ω^{-1} are used as unit length and time. The discrete values of $E_Z(n)$ are, in fact, ''energy per unit wave number,'' where ''unit wave number'' is set dimensionless, i.e., equal to one for each n making the units of $E_Z(n)$ equal to those of $(\Omega R)^2$. Such choice of units ensures that $E_Z(1) = C_Z$ for the theoretical spectrum (2). The long straight lines superimposed on the data are the theoretical -5 spectra, Eq. (2), that use the actual values of R and Ω for Jupiter and Saturn. [Here, $C_Z = 0.5$ rather than 0.3 (Ref. 3) was used as it gave a somewhat better agreement with data.] The short straight lines show the Kolmogorov spectra, Eq. (1), using $\epsilon \simeq 10^{-7}\ \mathrm{m}^2\,\mathrm{s}^{-3}$ for Jupiter as estimated from the data on heat flux intensity averaged over its surface.[6] For simplicity, the same value of ϵ was assigned to Saturn. The crossover wave number, n_{cr}, is obtained by equating Eqs. (1) and (2) giving $n_{\mathrm{cr}} \sim 100$ for both planets.

The spectra in Figs. 2(a) and 2(b) admit a simple phenomenological model. At the low wave number end, $n < n_{\mathrm{fr}}$, the spectra stay approximately level, $E(n) \simeq E(n_{\mathrm{fr}})$, which is consistent with the action of some form of a large scale friction. Careful analysis of the data gives the frictional wave numbers $n_{\mathrm{frJ}} \simeq 20$ for Jupiter and $n_{\mathrm{frS}} \simeq 10$ for Saturn. For $n_{\mathrm{fr}} < n < n_{\mathrm{cr}}$, the spectra steeply descend according to the -5 law, Eq. (2). The range $n > n_{\mathrm{cr}}$ behaves rather like white noise and is apparently unresolved.

The steep spectra are most pronounced over a band of intermediate wave numbers from about 20 $(= n_{\mathrm{frJ}})$ to 90 for Jupiter and 20 to 80 for Saturn (even though n_{frS} is better estimated as 10 for Saturn, there exists a transition zone between $n = 10$ and 20; a sharp drop of the spectrum with increasing n is more evident for $n > 20$). To further quantify the evidence for the -5 slope, a linear regression analysis was performed for the pairs $[\log n, \log E_Z(n)]$ for selected ranges of n. This analysis aimed to fit the data by a straight line, $\log E_Z(n) = A \log n + B$, that minimizes the variance, with A being the spectral slope and $C_Z = 10^B$ in Eq. (2). For Jupiter, regression based on the range $20 < n < 55$ gives a spectral slope of -4.7 ± 1.3 and $B = -0.77 \pm 2.0$ while for the range $20 < n < 90$, $A = -4.1 \pm 0.65$ and $B = -1.6 \pm 1.1$. For Saturn, the analysis based on a range $20 < n < 80$ gives a slope of $A = -4.8 \pm 0.75$ and $B = -0.52 \pm 1.26$. The estimates of the spectral slope are fairly close to -5 for both planets. These estimates provide information not only on spectral slopes but

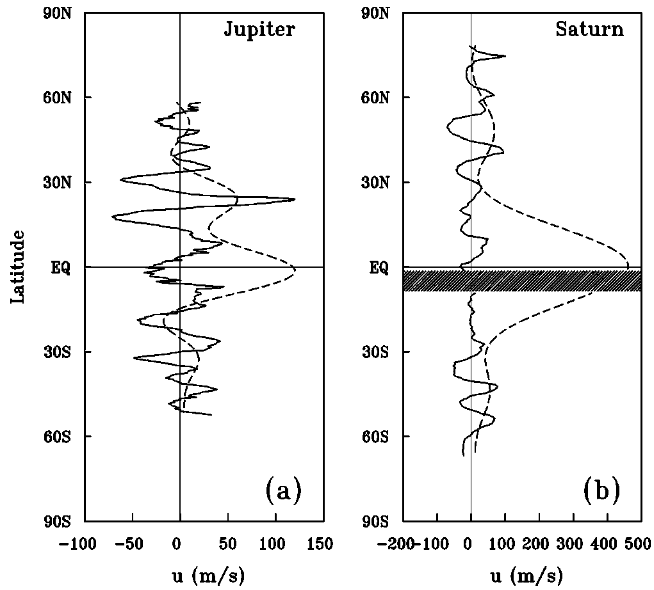

FIG. 3. Zonal wind profiles reconstructed from the first n_{fr} modes (dashed line) and the rest of the modes (solid line) for Jupiter (a) and Saturn (b). The shaded area for Saturn marks the equatorial region obscured by the ring.

also on their amplitudes. The linear regression analyses performed in various ranges of n show that when $A \to -5$, then $C_Z \in (0.3, 0.5)$, in agreement with our previous estimates.[3] The deviation of A from -5 causes the exponential change in the value of C_Z to compensate the error. For instance, $A = -3.5$ gives $C_Z = 0.003$. The reason for such a behavior is the close agreement between the theoretical line and the observational data. In other words, not only the spectrum of the data has a slope close to -5 but in addition, the spectral amplitudes fluctuate around their theoretical values. If flow regimes with the spectral slopes different from -5 were used to describe the data, they would have required values of C_Z unrealistically different from $O(1)$. The emergence of the steep -5 spectrum on a rotating sphere is accompanied by the energy accumulation in the large-scale zonal flows, in agreement with observations (according to Voyager data,[10] Jupiter's zonal kinetic energy is two to three orders of magnitude larger than its meridional counterpart) and computer simulations.[3] Summarizing, the spectrum (2) pertains to the only known physical mechanism that can explain the observational data; the conclusion reinforced by its good quantitative agreement with the observed spectra on both Jupiter and Saturn.

Consider now general characteristics of the Jovian/Saturnian atmospheres in the framework of the phenomenological spectral model. Integrating the idealized spectrum from 0 to ∞, a simple expression for the total kinetic energy is obtained,

$$\mathcal{E} = (5C_Z/4)(\Omega/R)^2 n_{\mathrm{fr}}^{-4}. \tag{3}$$

For this equation to be practically useful in a general case, one needs to know how to estimate n_{fr}. Observe that due to the steepness of the spectrum (2), the lowest modes in the -5 slope range carry most of the energy of that range and dominate the signature of the zonal wind profile which is directly related to the number of zonal jets. Therefore, n_{fr} can be roughly estimated from the prevailing number of zonal jets on a planet.

To further elucidate the physical meaning of n_{fr}, the zonal wind profiles were decomposed into components comprised of the modes $n < n_{\mathrm{fr}}$ and $n > n_{\mathrm{fr}}$ as shown in Figs. 3(a) and 3(b) for Jupiter and Saturn, respectively. Apparently, the first n_{fr} modes represent the equatorial jet while the modes $n > n_{\mathrm{fr}}$ account for the rest of the profile. This residual profile has the -5 spectral slope, is spaced more evenly than the full profile, and its equatorial jet is *westward*, particularly for Jupiter (for Saturn, the obscured by the ring equatorial band may have a significant effect on spectral decomposition). Generally, there is a good qualitative agreement between the computer simulated profiles[3] and observed profiles comprised of the modes $n > n_{\mathrm{fr}}$, both representing the -5 range of the spectrum.

It has been customary to scale the width of the zonal jets with the Rhines's length scale, $n_R^{-1} = (U/\beta)^{1/2}$, where U is the rms velocity fluctuation and β is the planetary vorticity gradient.[11] Substituting $U = (2\mathcal{E})^{1/2}$ from Eq. (3) and $\beta = \Omega/R$, one obtains $n_R \simeq n_{\mathrm{fr}}$. Thus, the frictional and Rhines's wave numbers appear to coincide demarcating the low wave number terminus of the spectral range with the -5 slope.

Golitsyn[1] estimated that although the rotation rates and radii of Jupiter and Saturn are very close, the total kinetic energy of the Saturn's atmosphere, $\mathcal{E}_{\mathrm{S}}$, is about an order of magnitude higher than that of Jupiter, $\mathcal{E}_J$. He found such a big difference "puzzling." Equation (3) helps to resolve Golitsyn's conundrum giving $\mathcal{E}_{\mathrm{S}}/\mathcal{E}_{\mathrm{J}} \simeq (n_{\mathrm{frS}}/n_{\mathrm{frJ}})^{-4} \simeq 2^4$. Similarly, for the ratio of the maximum velocities of the respective equatorial jets one obtains $U_{\mathrm{S}}/U_{\mathrm{J}} \simeq (n_{\mathrm{frJ}}/n_{\mathrm{frS}})^2 \simeq 2^2$, in agreement with the data in Fig. 1. The phenomenological model helped to narrow the problem down to one key parameter, n_{fr}. To explain the mystery of the 4-to-1 ratio, future research only needs to elucidate why the large scale friction acts on a wider spectral range on Jupiter than on Saturn.

Equation (3) indicates that the equatorial jets that reside in the frictional range $n < n_{\mathrm{fr}}$ contain about 80% of the total kinetic energy and appear to be huge capacitors of the kinetic energy of atmospheric circulations on giant planets; this interesting observation was also made by Golitsyn.[1] The physics of the equatorial jets must be distinguished from the "background" circulation that has the -5 spectrum. Golitsyn[1] emphasized that the nature of the equatorial acceleration is another central problem of the planetary sciences (in addition to the physics of the zonal circulation) that awaits resolution. We conjecture that the equatorial jets are the manifestation of the large scale energy condensation due to inverse energy cascade (similar to that in Smith and Yakhot[12] but for rotating, anisotropic turbulence) under the influence of large scale friction. This phenomenon could probably be reproduced in a barotropic model with an appropriate large scale drag. Generally, the details of the friction could prove to be crucial in determining the structure of the equilibrated zonal jets. More detailed information about the friction should be particularly welcome from future observations including the ongoing Cassini mission.

The primary balance of kinetic energy on a giant planet is $\mathcal{E}/\tau \simeq \epsilon$, where τ is the time scale of the large scale friction. Combining this relationship with Eq. (3) leads to $\Omega\tau = (5C_Z/4)(n_\beta/n_{\rm fr})^5 R n_{\rm fr}$, where $n_\beta = [(\Omega/R)^3/\epsilon]^{1/5} \approx 200$, giving τ of the order of 10^2-10^3 Earth's years for both planets. This time scale is longer than the approximately 100 year history of observations of the giant planets' atmospheres which underscores the necessity for an extremely long time average to smooth their observed energy spectrum of zonal mean flows.

The agreement between the n^{-5} spectrum in the barotropic 2D turbulence model and observations is encouraging. However, barotropic theories have some drawbacks. A well-known issue is that a forced barotropic model usually produces zonal jets that do not violate the Rayleigh–Kuo stability criterion while the observed zonal winds on Jupiter and Saturn do violate this criterion.[4,10,13] As a general remark, note that the Rayleigh–Kuo stability criterion is applied to the *mean* velocity profiles rather than the observed on the giant planets fields of slowly evolving transients that can be unstable. In addition, conventional barotropic model simulations, including our own, have used an idealized forcing (random but statistically uniform in space, white noise in time) and an even more idealized large scale drag (either a constant bottom drag or none at all). The possibility of simulating new flow regimes—including potentially unstable ones—by using more complicated forcing/dissipation in the barotropic model is severely underexplored. For example, the use of a more realistic forcing that imitates the recently discovered moist convection cells on Jupiter[5,6] would be worth considering. Note also that the Rayleigh–Kuo criterion is strictly valid for an unforced, inviscid, laminar zonal flow. While one can modify the stability criterion by going from barotropic to baroclinic (or shallow water) models,[4] one can also modify it within the framework of barotropic model by (say) incorporating friction. A flow profile that violates the inviscid Rayleigh–Kuo criterion may not violate the viscous Rayleigh–Kuo criterion with an external friction.[14] These possibilities remain to be explored.

Observe that the central to this paper -5 scaling law, (2), does not explicitly depend on the stability or instability of zonal jets. As long as the zonal energy spectrum depends only on the wave number and the gradient of planetary vorticity [n and Ω/R on the rotating sphere], dimensional analysis leads to the -5 spectral slope regardless of the underlying dynamics (this was pointed out by Rhines,[11] though for Rossby waves' energy spectrum). Thus, the stability or instability of the jets may not necessarily be crucial to their spectral properties, particularly for flow regimes in which the second derivative of the zonal wind has the order of magnitude of Ω/R.

The barotropic model used in our previous simulations[3] excludes the effect of finite radius of deformation, L_d, estimated between 500 and 2500 km for Jupiter's troposphere.[4] The corresponding wave numbers occupy the range $n = \pi R/L_d \in (90,400)$ which is not well resolved in the data. By focusing on the range $n < 90$, this study covers the scales larger than L_d, such that $nL_d < 1$. It is generally recognized that, on a fast-rotating planet, the 3D quasigeostrophic turbulence behaves like barotropic 2D turbulence at least on the intermediate-to-large scales with $L > L_d$,[15] which thus justifies the use of the barotropic model in the present study.

Turbulent flows possess only a few universal spectra that are reliably observable. Among them are the Kolmogorov's $-5/3$ spectrum in 3D and 2D turbulence and Kraichnan's -3 spectrum in the enstrophy cascade range of 2D turbulence. Our newly discovered -5 spectrum pertains to quasi-one-dimensional turbulence. The appearance of this spectrum on Jupiter and Saturn indicates that it may indeed be common in Nature with C_Z in Eq. (2) being a universal constant.

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